1 Consider the function $f(x,y) = 4 - \sqrt{x^2+y^2}$

a) Sketch the graph $z = f(x,y)$

b) Calculate $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$

c) what is the domain of f? of $\frac{\partial f}{\partial x}$? of $\frac{\partial f}{\partial y}$?

b) where is f continuous? $\frac{\partial f}{\partial x}$? $\frac{\partial f}{\partial y}$?

c) Show that the limit $\lim\limits_{(x,y)\to(0,0)} \frac{\partial f}{\partial x}$ does not exist

d) Show that the limit $\lim\limits_{(x,y)\to(0,0)} x^2\frac{\partial f}{\partial x}$ exists.

e) At what points is f differentiable?

f) Write the differential df.

h) Write the equation of the tangent plane to the graph of f at the point $P_0(1,0,3)$

j) Write the linear approximation of f at this point P_0.

2. Consider the function $g(x,y) = x \ln(x+y)$
Calculate the rate of change $\frac{dg}{dt}$ at $t=0$
when $x = t^2+1$, $y = 2t$.

3. Let $z = h(x,y)$ be a function with its second derivatives continuous. If $x = p^2+q^2$, $y = p-q$ find $\frac{\partial^2 z}{\partial p \partial q}$.

4. Show that the function $u(x,t) = e^{-4t}\sin(2x)$ satisfies the heat equation $u_t = c\,u_{xx}$ for some value of the constant c and find this constant

5. Let $h(t)$ be a differentiable function. Assuming that the equation $h(x-z)+h(y-z)=1$ defines implicitly z as a function of x and y find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

6. Find the maximum rate of change of

$$f(x,y,z) = xy\,e^{yz} + x + yz$$

at the point $(1,2,0)$ and the direction in which it occurs.